# CS 170 Homework 1

Due **Saturday 2/1/2025, at 10:00 pm (grace period until 11:59pm)**

## 1 Study Group

List the names and SIDs of the members in your study group. If you have no collaborators, explicitly write "none".

## 2 Course Policies

(a) What dates and times are the exams for CS170 this semester? Are there planned alternate exams?

(b) Homework is due Saturdays at 10:00pm, with a late deadline at 11:59pm. At what time do we recommend you have your homework finished?

(c) We provide 2 homework drops for cases of emergency or technical issues that may arise due to homework submission. If you miss the Gradescope late deadline (even by a few minutes) and need to submit the homework, what should you do?

(d) What is the primary source of communication for CS170 to reach students? We will send out all important deadlines through this medium, and you are responsible for checking your emails and reading each announcement fully.

(e) Please read all of the following:

(i) **Syllabus and Policies:** `https://cs170.org/policies/`

(ii) **Homework Guidelines:** `https://cs170.org/resources/homework-guidelines/`

(iii) **Regrade Etiquette:** `https://cs170.org/resources/regrade-etiquette/`

(iv) **Forum Etiquette:** `https://cs170.org/resources/ed-etiquette/`

Once you have read them, copy and sign the following sentence on your homework submission.

"I have read and understood the course syllabus and policies."

## 3 Understanding Academic Integrity

Before you answer any of the following questions, make sure you have read over the syllabus and course policies (`https://cs170.org/policies/`) carefully. For each statement below, write *OK* if it is allowed by the course policies and *Not OK* otherwise.

(a) You ask a friend who took CS 170 previously for their homework solutions, some of which overlap with this semester's problem sets. You look at their solutions, then later write them down in your own words.

(b) You had 5 midterms on the same day and are behind on your homework. You decide to ask your classmate, who's already done the homework, for help. They tell you how to do the first three problems.

(c) You're a serial procrastinator and started working on the homework at 9:00 PM on Friday, and out of desperation searched up a homework problem online and find the exact solution. You then write it in your words and cite the source.

(d) You were looking up Dijkstra's on the internet, and inadvertently run into a website with a problem very similar to one on your homework. You read it, including the solution, and then you close the website, write up your solution, and cite the website URL in your homework writeup.

(e) You're almost done with this week's homework, but there's this one small detail of an algorithm that you just can't get. To avoid wasting time going to office hours, you decide to ask ChatGPT how to complete the algorithm. You write their response in your own words and cite ChatGPT.

# 4 Math Review

The following subparts will cover several math identities, tricks, and techniques that will be useful throughout the rest of this course.

(a) Simplify the following expressions into a single logarithm. Your answer should be in the form $\log_a b$ or $\ln(b)$:

(i) $\frac{\ln x}{\ln y}$

(ii) $\ln x + \ln y$

(iii) $\ln x - \ln y$

(iv) $170 \ln x$

(b) Give a simple proof for each of the following identities:

(i) $x^{\log_{1/x} y} = \frac{1}{y}$

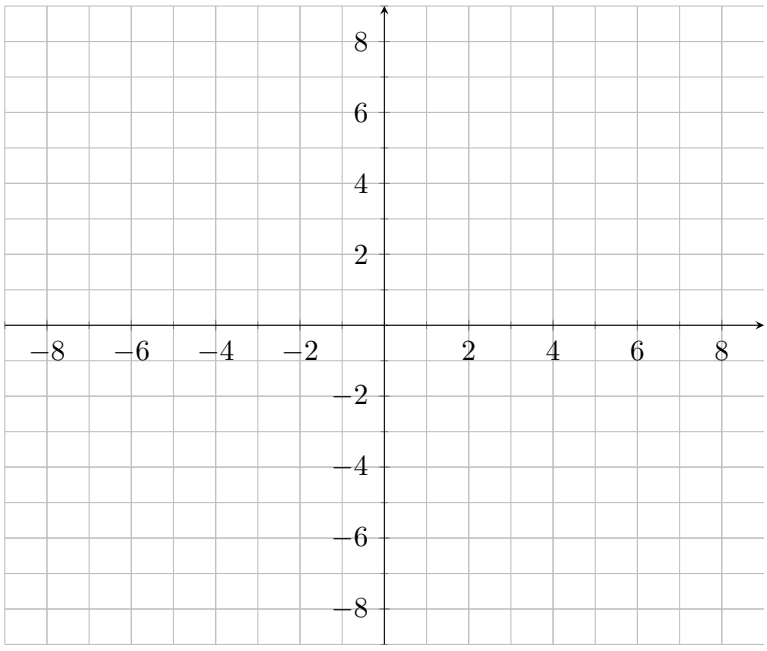
(ii) $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

(iii) $\sum_{k=0}^{n} ar^k = \begin{cases} a\left(\frac{1-r^{n+1}}{1-r}\right), & r \neq 1 \\ a(n+1), & r = 1 \end{cases}$

(c) Consider the following two inequalities:

$$x + 3y \leq 6$$
$$y \geq 2x - 5$$

Graph out the set of all points $(x, y)$ that satisfy both inequalities above. Clearly mark your lines and shaded region(s).



# 5 Asymptotics Practice

(a) For each pair of functions $f$ and $g$ below, specify whether $f = O(g)$, $g = O(f)$, or both. No justification needed.

| $f$ | $g$ | $f = O(g)$ | $g = O(f)$ | Both |
|---|---|---|---|---|
| $n^2 + 5n$ | $1000(n+1)^2$ | ◯ | ◯ | ◯ |
| $5n^3$ | $n^3 + (\log n)^{10}$ | ◯ | ◯ | ◯ |
| $n^{100}$ | $1.01^n$ | ◯ | ◯ | ◯ |
| $(\log n)^{100}$ | $n^{0.1}$ | ◯ | ◯ | ◯ |
| $n \cdot 2^n$ | $3^n$ | ◯ | ◯ | ◯ |
| $n!$ | $n^n$ | ◯ | ◯ | ◯ |

(b) Consider the two functions $f(n) = 1 + b + b^2 + \cdots + b^n$ and $g(n) = b^n$ for an arbitrary constant $b > 0$. Similar to what you did in the previous part, specify whether $f = O(g), g = O(f)$, or both. **Justify your answer.**

*Hint: the asymptotic relationship between $f$ and $g$ varies depending on the specific value of $b$.*

# 6 Recurrence Relations

For each part, find the asymptotic order of growth of $T(n)$; that is, find a function $g$ such that $T(n) = \Theta(g(n))$. Show your reasoning and **do not directly apply the Master Theorem; doing so will yield 0 credit**.

In all subparts, you may ignore any issues arising from whether a number is an integer.

(a) $T(n) = 2T(n/3) + 5n$

(b) An algorithm $\mathcal{A}$ takes $\Theta(n \log n)$ time to partition the input into 5 sub-problems of size $n/5$ each and then recursively runs itself on 3 of those subproblems. Describe the recurrence relation for the run-time $T(n)$ of $\mathcal{A}$ and find its asymptotic order of growth.

(c) $T(n) = T(3n/5) + T(4n/5)$ (We have $T(1) = 1$)

*Hint: first, compute a reasonable upper and lower bound for $T(n)$. Then, try to guess a $T(n)$ of the form $an^b$ and then use induction to argue that it is correct.*

(d) $T(n) = T(\sqrt{n}) + 1$ (You may assume that $T(2) = T(1) = 1$)

*Hint: perform a change of variables.*